

**Predicting LIGO's (the Laser Interferometer Gravitational-  
Wave Observatory) future performance using past data**

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## **ABSTRACT**

LIGO's future performance can be predicted using past data. By measuring the decrease of ASD over S5, S6, and O1, a trend line can be drawn to show the past improvements of LIGO and to predict the future trend. A rough prediction of whether LIGO can detect the weak primordial wave signal in the near future is given based on its frequency range and strain amplitude.

## INTRODUCTION

LIGO's past improvements could be used to predict its future performance. In order to determine the scale of its previous improvements, I figured out how the smallest strain amplitude in a specific frequency range decreases over S5, S6, and O1. These data points could be drawn in one graph as a curve, and this curve can be used to predict the future trend of improvements.

## **BACKGROUND**

### **The general introduction of Gravitational Waves**

Gravitational Waves are time-varying oscillations of the gravitational field. They are typically caused by the changes in space-time produced by a moving mass, and they propagate out as transverse waves, travelling at the speed of light. In fact, every moving object which has a changing quadrupole moment (however minute it is) can cause the radiation of gravitational waves, but due to the limitations of our most sensitive detectors, only the most significant ones can be detected by us. For example, binary systems including the collisions and coalescences of neutron stars and black holes (and the final state called a “ring-down”), supernova explosions of massive stars (which might consequently form neutron stars and black holes), accreting neutron stars, rotations of neutron stars with deformed crusts, gravitational radiation created by the birth of the universe, etc. can all radiate considerable amount of gravitational waves still detectable to

us even after the long journey through the dust and particles in the universe. Additionally, gravitational waves act as “tensors”, the quadruple distortions of space-time which travel like some soft sea creatures: when two different transverse directions come closer, the other two fall further off from each other. As this property suggests, gravitational waves are direct evidence that space and time should be combined in astronomy, put into consideration as “space-time”, and they can act as a whole in operating our delicate universe. [1]

### **The principles and enhancements of LIGO.**

The goal of the Laser Interferometer Gravitational-Wave Observatory (LIGO) was to directly detect gravitational waves through the interference pattern two separate light beams create. When there are no gravitational waves to compress or stretch space-time, the single light beam emitted by one laser at the far end of the tunnel will get split up by a half-silvered mirror, going their separate ways in two directions and being respectively reflected back by two

other mirrors to converge into one beam again, thus (with their initial frequency and phase difference) creating an interference pattern (the property of all kinds of waves). However, if there are gravitational waves passing through the space-time around the detector, they will distort everything inside, including wavelengths. After being compressed or stretched out by the distorted space-time, the two interfering light will not exactly offset or intensify each other in the presumed dark and light stripes region; instead the interference pattern will look more irregular with wider dark stripes in some regions and narrower light regions in others. Scientists used “strain amplitude” ( $h$ ) as the sensitivity of detecting gravitational waves. The definition of “ $h$ ” :

$$h = \frac{\Delta L}{L}$$

where  $L$  is the length of one arm of the interferometer, and  $\Delta L$  is the change in that length. The size of “ $h$ ” is typically small so the equipment must reach a high sensitivity in order to meet the

demanding requirements of precisely filtering out the background noise. The huge limitations of LIGO include the inevitable detection of fake signals, such as the seismic noise (including logging noise and even the slam of a door) at low frequency, thermal noise (the atomic vibrations) inside the components at middle frequency, and “shot noise” (the quantum nature of light) at high frequency. Determined to enhance the sensitivity of LIGO, operators and engineers worked collectively to seek out new ways to upgrade the present interferometers. In 2007 they installed e-LIGO, the short form of “enhanced LIGO”, which added an active seismic isolation system in LLO Louisiana (LIGO Livingston Observatory), an output mode cleaner that can effectively filter out the unnecessary data, and an in-vacuum readout HW. In 2011 the “Adv-LIGO” (the advanced LIGO) was put into construction, which added another active seismic isolation system in LHO (LIGO Hanford Observatory), Washington. Just a few days after its new improvements, the Adv-LIGO detected

the signal from the merging of two colliding black holes of 30 solar mass, and the dream of receiving the “calls” from significant events in our universe was finally realized. [2]

### **The general introduction of primordial waves**

About 13.8 billion years ago, our universe came into being. The significant event called “the Big Bang” [3] created our universe in about  $10^{-35}$  seconds. And within a trillionth of a trillionth of a trillionth of a second, the universe underwent dramatic changes, including the explosive growth and dramatic increase of temperature. During the expansion of the universe at the beginning of the cosmic dawn, gravitational waves were possibly created. These vestigial traces of the early universe have so far travelled for nearly 14 billion years and they have been gradually diluted over time. The Cosmic Gravitational Wave Background (CGWB), as they are called, offers a powerful way to study the cosmos, and astronomers have scrutinized it with steadily mounting precision ever since its discovery a half-century ago. [4]

In 2014, a group of researchers of Background Imaging of Cosmic Extragalactic Polarization (BICEP) thought they had found evidence of gravitational waves created immediately after the Big Bang: a swirling “B-mode” pattern of light. Unfortunately, it later turned out to be merely galactic dust. [5]

Since the magnitude of gravitational waves produced by inspiraling black holes or neutron stars is much larger than those in the CGWB, it might be much easier for LIGO to look for GW sources from interacting black holes. As technology progresses, scientists might be able to detect weak signals like GW from CGWB by using upgraded Active Seismic Isolation system, more powerful laser interferometers, and more available data channels.

## RESEARH PROCESS

### **1 Choose a time period that contains the most science mode data for S5, S6 and O1.**

LIGO uses UTC time and GPS time to indicate the time period during which the data is recorded. H1 is located at Washington State, which uses PST time 8 hours later than UTC. L1 is located at Louisiana, which uses CST time 6 hours later than UTC. Therefore, when I am downloading data (which uses UTC time) from both L1 and H1, I have to subtract 8 hours for H1 and 6 hours for L1 to determine their separate time periods.

For L1, I choose the data from 5:00-6:00 a.m. UTC period for the following three reasons.

First, Livingston, Louisiana uses CST for its time zone, and 5:00-6:00 a.m. UTC equals 11:00-12:00 p.m. which is midnight for Livingston. During midnight, people go to sleep so there is much less human noise (e.g. logging, driving, or even opening a door)

interfering LIGO detectors' performance.

Second, I found that nearly every day during 5:00-6:00 a.m. UTC LIGO detectors are at their science mode, so I can pick out several consecutive days and measure LIGO's performance more consistently.

Third, during this period the data files all contain 100% science mode data, which means I can use all the data in the files as the materials for my research.

For H1, I also choose the data from 5:00-6:00 a.m. UTC period. Even if this may not equal to the quietest midnight period in Hanford, Washington, it's still evening period so human activities may not have an influential impact on detectors' overall performance. Additionally, I want to compare the performance of L1 and H1, so I have to choose the same time period in order to control the variables. Therefore, the time period 5:00-6:00 a.m. UTC may be the best option.

## **2. Find a way to comprehensively represent LIGO's performance**

**over time.**

For each site, H1 and L1 (I wipe out H2 because later on it was shut down), I divide its operation period into three main parts: Initial Period, Middle Period, and End Period. For each period, I find five consecutive days which (around 5:00 a.m. to 6:00 a.m.) all contain 100% science mode data. As a result, for H1 of S5, I download 15 data files (3 periods\*5 data files per period), and the same is for L1. So for each science run, 30 files are downloaded. Since I want to see LIGO's improvements over S5, S6, and O1, I download 90 data files in total.

### **3. Choose an appropriate time to download data files.**

I did research on when is the best time to download data files from <https://losc.ligo.org/data/> in mainland China. I divided 24 hours into 6 separate parts and explored the downloading speed at 6 specific times: 0:00, 4:00, 8:00, 12:00, 16:00, and 20:00. The result is that at around 4:00 p.m. CST (China Standard Time) the downloading speed

is at its peak, partly because during this period people in the United States have not woken up so the connection of VPN in mainland China is much better than in other time periods.

Additionally, it's a better time to download data files during midnight (from 0:00 to 3:00 a.m.), but I usually fall asleep at that time. Therefore, I want to command my computer (Mac Pro) to automatically download the data using command lines in Terminal. Here is the procedure taught by professor Eric Myers, and it may only apply to Mac. [6]

First, to test the system, we need to input the command “at” followed by a time or date (or both). The command “atq” will show us the contents of the queue of commands waiting to be run.

Second, input “cd Desktop” to change directory to Desktop, so when the downloading process is finished, the data file will automatically appear on the desktop.

Third, input “curl -L -O -s” followed immediately by a blank

space and the URL of the data file. The URL can be obtained by right click the “hdf5” format of the file on the LOSC website and choose “copy the URL”. By default, curl will output whatever it downloads to “standard output”, so to cause it to save the data to a file of the same name I will add the “-O” flag (that is a capital letter Oh). And since that URL is not the real URL for the file, it results in a redirect. Therefore, I also include the “-L” flag to tell curl to follow the redirect. Also, curl can be very verbose while it operates (it will display download status). But for my purpose I just want it to work silently, so I’ll add the “-s” flag. Or if you want to report the downloading progress, you can omit the “-s” part.

Fourth, control C can stop the run, and control D will show a line telling me the job number and date and time it will run. The command “atq” will show me the contents of the queue of commands waiting to be run. The output is sent to me using the Unix mail system. It will not go to my gmail account, but I can easily get it using the

local Unix “mail” command.

Finally, the data file will appear in complete format on my desktop.

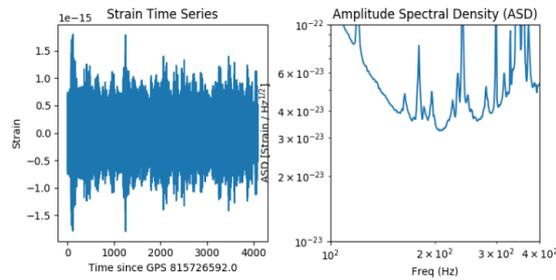
#### **4. Plot the ASD.**

The Power Spectral Density (PSD) is a representation of how the power in the data is distributed in frequency space.

The ASD can be obtained by taking the square root of the PSD. It can measure the strain amplitude “h” at different frequencies, which is exactly what I want for my research project. Here is how I plot the ASD using python scripts.

First, I created a directory named “Slot”. Next, I added the data file (in the format “.h5py”) I’ve downloaded from <https://losc.ligo.org/data/>, `readligo.py` (also downloaded at [https://losc.ligo.org/s/sample\\_code/readligo.py](https://losc.ligo.org/s/sample_code/readligo.py) ), and the script “`plot_strain.py`” into the “Slot” directory. Then I run the script and at

the end it will automatically plot a graph of ASD and save this graph



in the "Slot" directory in the format of ".PNG".

Fig.1

The "Slot" directory I've created.

Fig 1 shows what the final graph looks like by using the "H-H1\_LOSC\_4\_V1-815726592-4096.hdf5" data file.

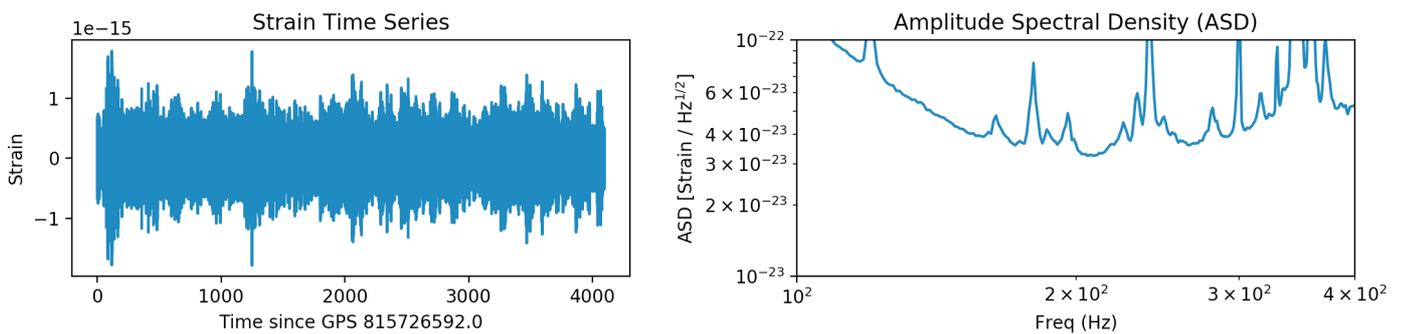


Fig.2 Left: time series graph. Right: ASD graph.

Actually the original script is downloaded from

<https://lsc.ligo.org/tutorial05/>, and the frequency range is 10 to 2000.

But I find that nearly all of the x-axis of the lowest point is among the 100 to 400 frequency range, so I modify the scripts and change its frequency range into 100 to 400. Also, the y-axis of all the lowest data points are densely distributed at the  $10 \text{ e-}23 \sim 10 \text{ e-}17 / \text{Hz}^{1/2}$ , so I change the scale of the y-axis into  $10 \text{ e-}23 \sim 10 \text{ e-}17$  (Like a zoom-in effect in Figure 3.)

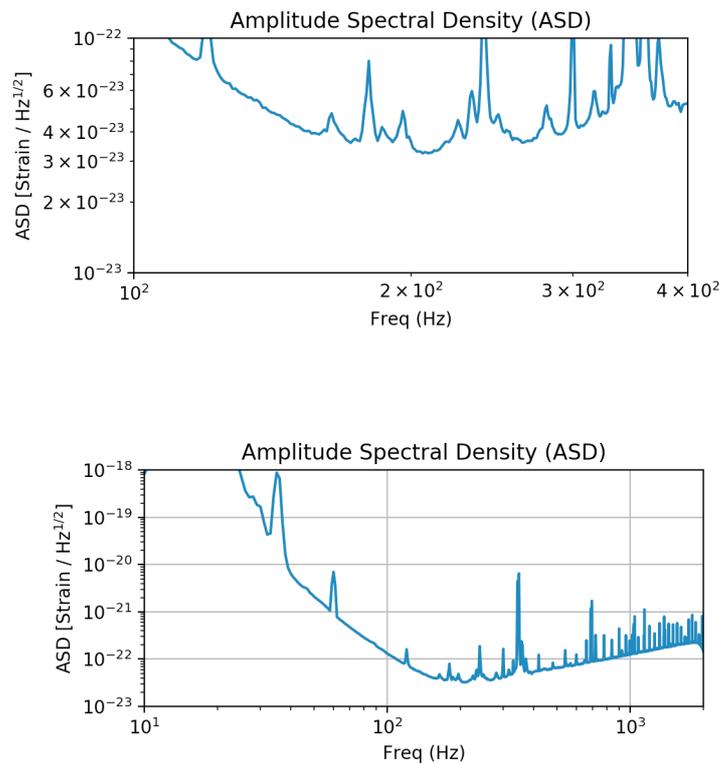


Fig.3

Up: the ASD graph using the original script.

Down: the ASD graph using the modified script.

Sometimes the chosen data files may be imperfect because the graph generated from them are far different from others. For example, the sweet-spot frequency corresponding to the lowest Amplitude Spectral Density may be much larger than 400 hertz. However, in this case this data point should not be deleted in order to maintain the authenticity of data.

## ANALYSIS

### 1. Read in the data.

After all the necessary ASD graphs were plotted, I magnified them and measured the lowest point in each graph using rulers and eyes. Also, I recorded the frequency of the lowest point. This might be a rough process, and might contain certain errors, but I tried to lower that error by approximating the data to the nearest hundreds. Then I input all the data points into Excel, and calculated the average ASDs of the Initial, Middle, and End Period of L1 and H1 in S5 (It is easier to type in the “Average” formula which automatically calculates the average value of chosen data). The average performance of LIGO over S5 should include both L1 and H1’s performance.

## 1) About the ASD

For S5, the lowest ASD of L1 and H1 is  $3.655\text{E-}23$  for the Initial Period,  $2.83\text{E-}23$  for the Middle Period, and  $2.625\text{E-}23$  for the End Period. Therefore, there is a steady decrease in the lowest ASD over S5, indicating that LIGO does improve over S5. Additionally, the standard deviation of ASD for Initial, Middle, and End period is  $4.148125\text{E-}24$ ,  $1.355749\text{E-}24$ , and  $1.0233145\text{E-}24$ . There may be more fluctuations for the Initial Period than for the End Period, because at its first run the detector may not be stable enough to collect data consistently.

As for S6, the lowest ASD of L1 and H1 is  $2.825\text{E-}23$  for the Initial Period,  $2.14\text{E-}23$  for the Middle Period, and  $1.935\text{E-}23$  for the End Period, also indicating an improvement over time. Additionally, the standard deviation of ASD for Initial, Middle, and End Period is  $3.252675\text{E-}24$ ,  $2.420475\text{E-}24$ , and  $1.223584\text{E-}24$ . This data set does also indicate that the detectors' performance grow stable over time.

As for O1, the lowest ASD of L1 and H1 is  $7.735\text{E-}24$  for the Initial Period,  $7.515\text{e-}24$  for the Middle Period, and  $7.755\text{e-}24$  for the End Period. The standard deviation of ASD for Initial, Middle, and End Period is  $2.148925\text{E-}25$ ,  $1.98903\text{E-}25$ , and  $3.98499\text{E-}25$ . The ASD of the End Period of O1 is not stable as seen from its large standard deviation, compared to the Initial Period and Middle Period. This is partly because the data files I chose contained certain biased or imperfect data. Or it's possible that something happened to LIGO during that period of time which severely impacted the detector sensitivity.

S6	Site	Period	Timeline	Average Timeline	UTC	Mininum ASD	Average for one cycle in one site	Standard Deviation	sweetspot(Frequency)	Average	Standard Deviation
H1	Initial	931323904	931828531.2	2009-07-11T05:04:49	2.55E-23	2.61E-23	5.11791E-24	175	200	42.72001873	
		931766272		2009-07-16T07:57:37	2.25E-23			180			
		931930112		2009-07-18T05:28:17	2.33E-23			195			
		932020224		2009-07-19T06:30:09	3.50E-23			275			
		932102144		2009-07-20T05:15:29	2.40E-23			175			
	Middle	951369728	951646617.6	2010-02-28T05:21:53	1.85E-23	2.09E-23	4.18903E-24	200	255	92.60129589	
		951455744		2010-03-01T05:15:29	1.85E-23			220			
		951717888		2010-03-04T06:04:33	1.95E-23			220			
		951803904		2010-03-05T05:58:09	2.83E-23			420			
		951885824		2010-03-06T04:43:29	1.95E-23			215			
	End	970895360	971225497.6	2010-10-12T05:09:05	1.80E-23	1.78E-23	5.63028E-25	185	181	2.236067977	
		970981376		2010-10-13T05:02:41	1.83E-23			180			
		971329536		2010-10-17T05:45:21	1.70E-23			180			
		971415552		2010-10-18T05:38:57	1.75E-23			180			
		971505664		2010-10-19T06:40:49	1.83E-23			180			
L1	Initial	931500032	931776921.6	2009-07-13T06:00:17	3.00E-23	3.04E-23	1.38744E-24	170	185	11.18033989	
		931590144		2009-07-14T07:02:09	3.20E-23			200			
		931672064		2009-07-15T05:47:29	2.85E-23			190			
		932020224		2009-07-19T06:30:09	3.00E-23			180			
		932102144		2009-07-20T05:15:29	3.15E-23			185			
	Middle	951373824	951649075.2	2010-02-28T06:30:09	2.10E-23	2.19E-23	6.5192E-25	190	184	4.183300133	
		951455744		2010-03-01T05:15:29	2.15E-23			180			
		951717888		2010-03-04T06:04:33	2.25E-23			180			
		951803904		2010-03-05T05:58:09	2.25E-23			185			
		951894016		2010-03-06T07:00:01	2.20E-23			185			
	End	970895360	971069030.4	2010-10-12T05:09:05	2.10E-23	2.09E-23	1.88414E-24	200	188	23.87467277	
		970981376		2010-10-13T05:02:41	2.05E-23			210			
		971071488		2010-10-14T06:04:33	2.40E-23			150			
		971157504		2010-10-15T05:58:09	1.90E-23			200			
		971239424		2010-10-16T04:43:29	2.00E-23			180			

Fig.4

The data set of S6.

Additionally, in order to comprehensively determine whether the datasets form a curve or a straight line, I have to download more data between Initial Period, Middle Period, and End Period. The process is the same as mentioned before, so I only present the final result in step 6 “Analyze the data”.

## **2) About the sweet-spot frequency**

I want to find at what frequency LIGO can detect the most minuscule signal.

For S5, the average sweet-spot frequency is 173.5 Hz; for S6, it's 198.83 Hz; for O1, it's 178.375 Hz. Therefore, at around 170-200 Hz, LIGO detectors can detect the weakest signal radiating from the faraway celestial objects.

## **2. Analyze the data**

I put all the data points into one graph and add an error bar which is the average standard deviation of all the data. Figures below can show how the data varies and determine whether LIGO grows stable over time.

## 1) The pattern of S5.

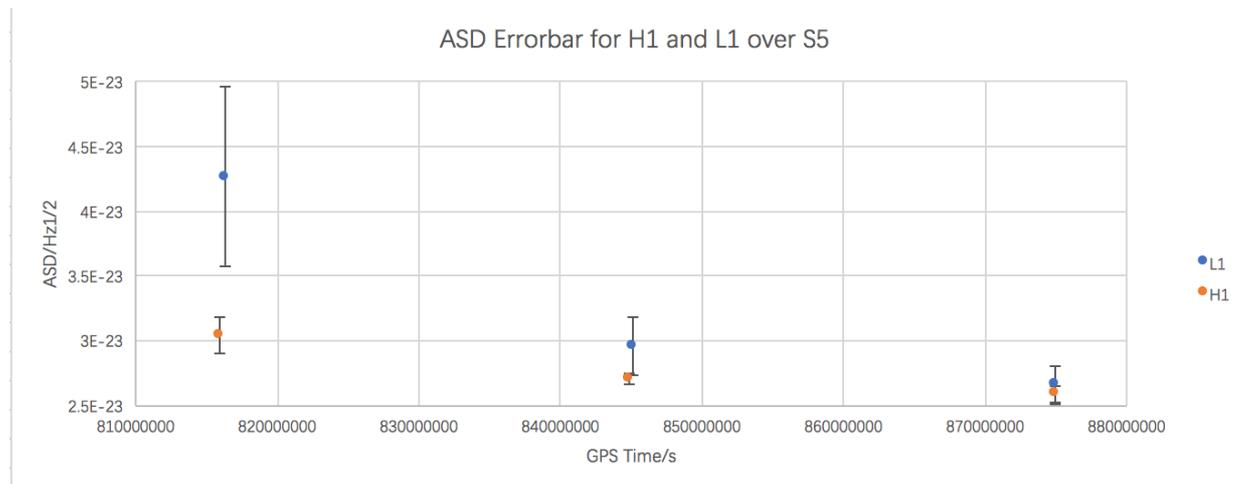


Fig.5

ASD with Error bar for H1 and L1 over S5

The overall trend of ASD of S5 indicates that at the end of S5 LIGO is near its best at that time. Over the course of S5, the curve of ASD is gradually bottoming out, which shows that after a significant upgrade before the start of S5, LIGO improves incrementally every day. At the end of S5, it reaches the best it can do after the upgrade, so the curve grows flat.

Then I downloaded more data between the periods and added more

data points to create a reliable trend line used to estimate the function of those data points. I supposed the function of the graph would be a decaying exponential. And here are the results.

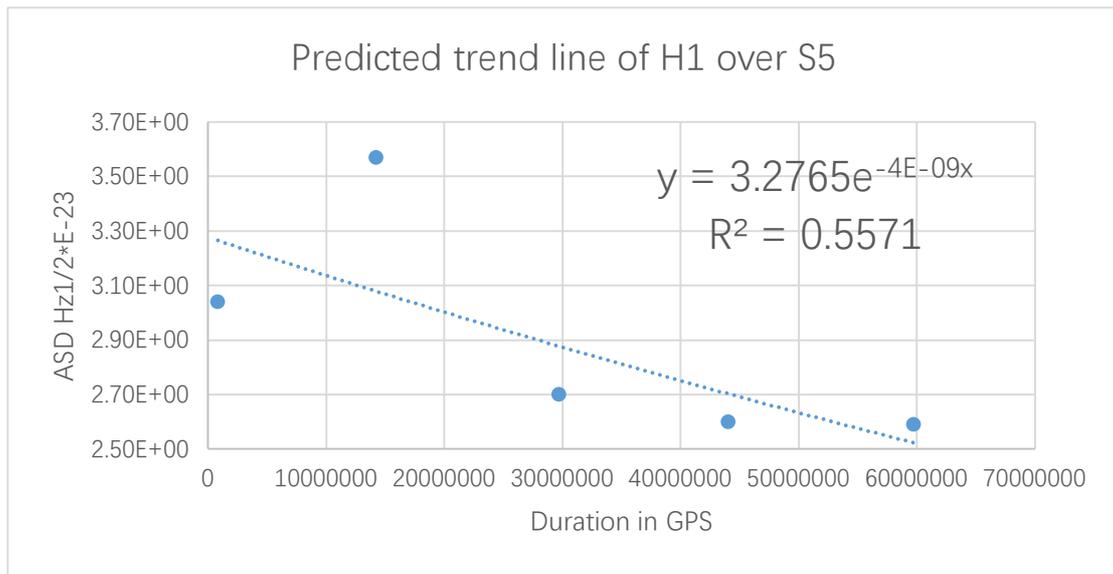


Fig.6

Predicted trend of H1 over S5.

One thing worth noting is that when I tried to add more data between Initial Period and Middle Period, all the data points were way outside the normal range. I wondered if something happened during this time period, resulting in a plummet of detector sensitivity. However, in order to investigate this idea, I omitted this data point.

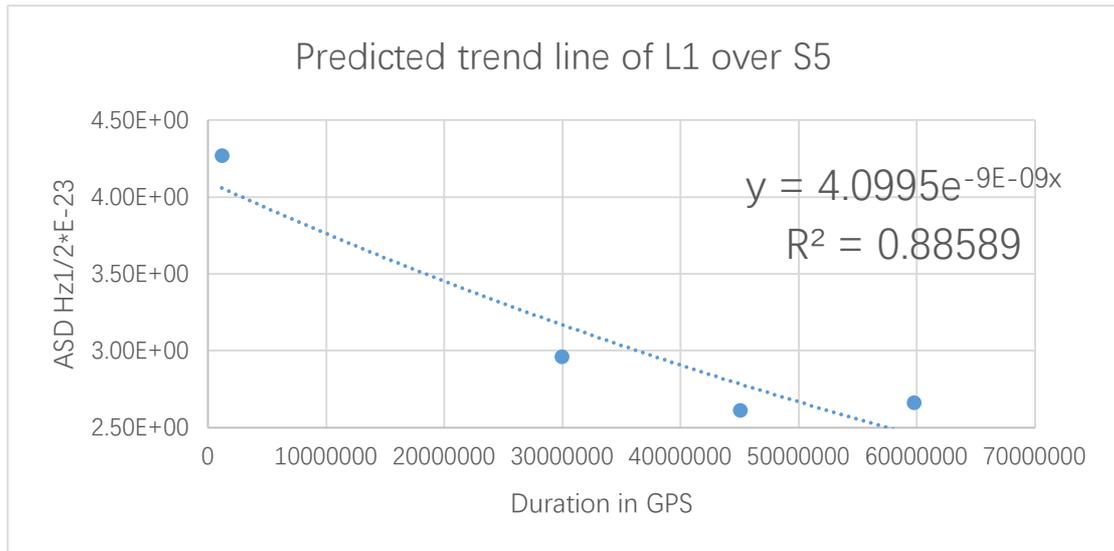


Fig.7

Predicted trend line of L1 over S5

The function for H1 over S5 is  $y = 3.2765e^{-4E-09x}$ , and the function for L1 over S5 is  $y = 4.0995e^{-9E-09x}$ . If we rewrite the exponent of e to  $-x/(\text{tau})$ , then the “tau” for H1 and L1 is 2.5E8 and 1.1E8, respectively. The scale of the y values is different from the initial data points, but that only changes the value of the amplitude rather than “tau”. It’s easier to plot a graph using the changed y values.

## 2) The pattern of S6

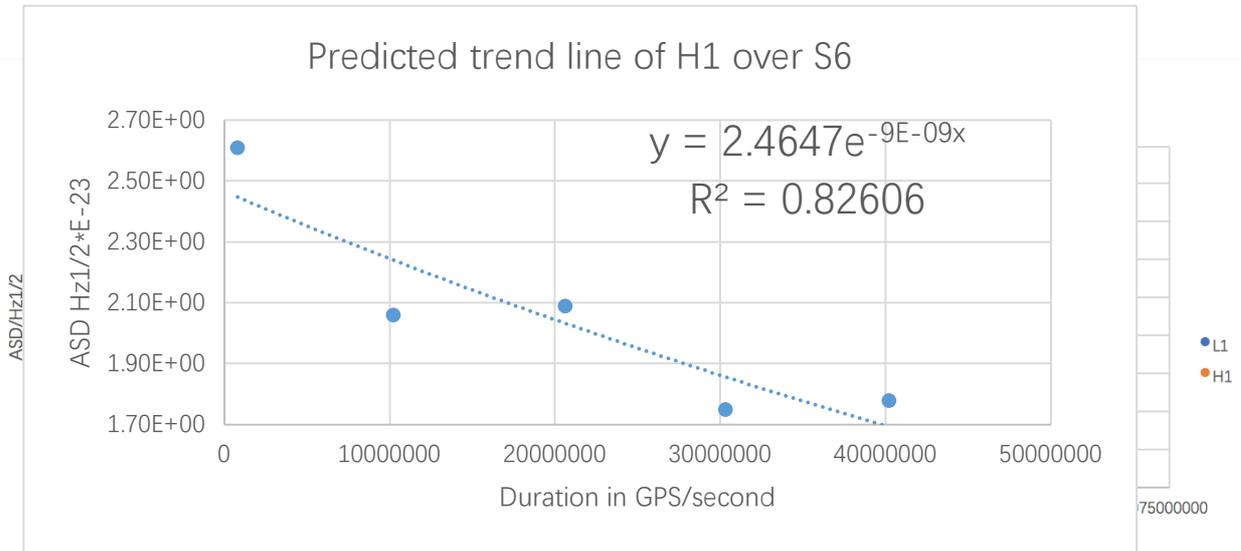


Fig.8

ASD with error bar for H1 and L1 over S6.

The overall trend of S6 is different from that of S5, and within this run the trend for H1 and L1 is different from each other. For L1, the trend of ASD is similar to that of S5: the curve becomes flattened out at the end of the run, indicating that LIGO is approaching its best after a major upgrade on its facilities. However, for H1, the pattern over S6 is more like a straight line, indicating that the improvements are stable over time. Another possibility is that the trend is actually

more like a curve, but with a steep slope at the beginning, and the detectors are shut down before they can achieve their maximum sensitivity after a major upgrade.

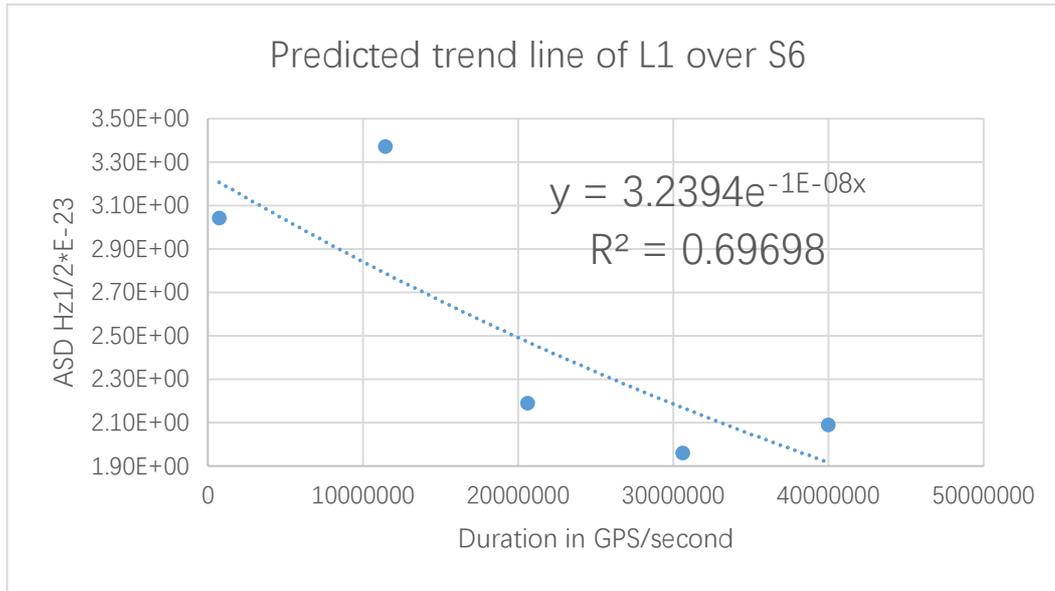


Fig.9

Predicted trend line of H1 over S6.

After I had added more data points between Initial Period, Middle Period, and End period, I found a trend line of those data points using an exponential decaying function. The function for H1 over S6 is  $y = 2.4647e^{-9E-09x}$ , and for L1 over S6 it's  $y = 3.2394e^{-1E-08x}$ . The “tau” value mentioned before is 1.1E8 for H1 and 1E8 for L1.

### 3) The pattern of O1

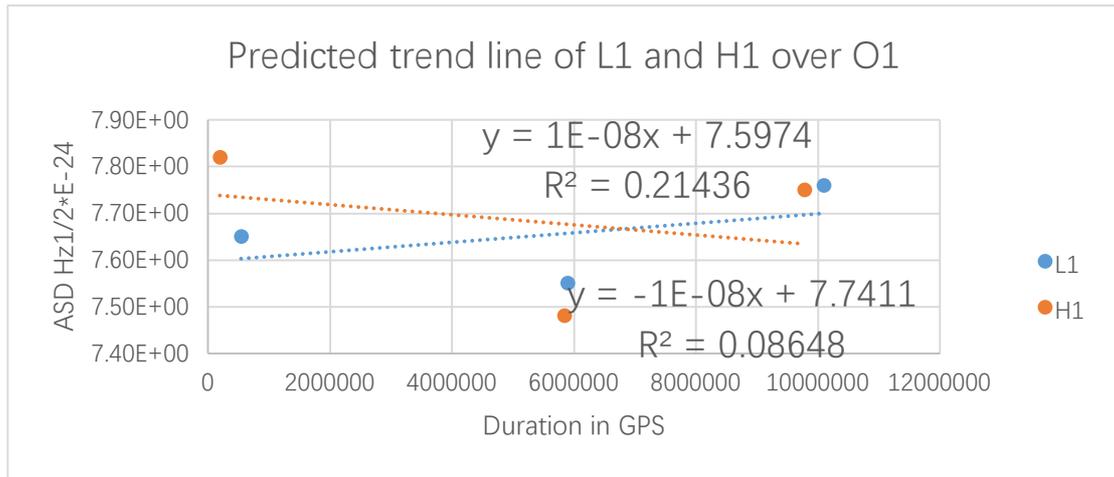


Fig.10

Predicted trend line of L1 and H1 over O1.

The pattern of O1 is more like a straight line, rather than an exponential decaying function. The data points are quite scattered, so I cannot see a clear exponential pattern over this period. The function for L1 is  $y = -1E-08x + 7.7411$ , and for H1 is  $y = 1E-08x + 7.5974$ . The slopes of these two lines are in a minuscule scale, indicating that the performance of LIGO during O1 is quite stable.

#### 4) The ASD connection between S5, S6, and O1

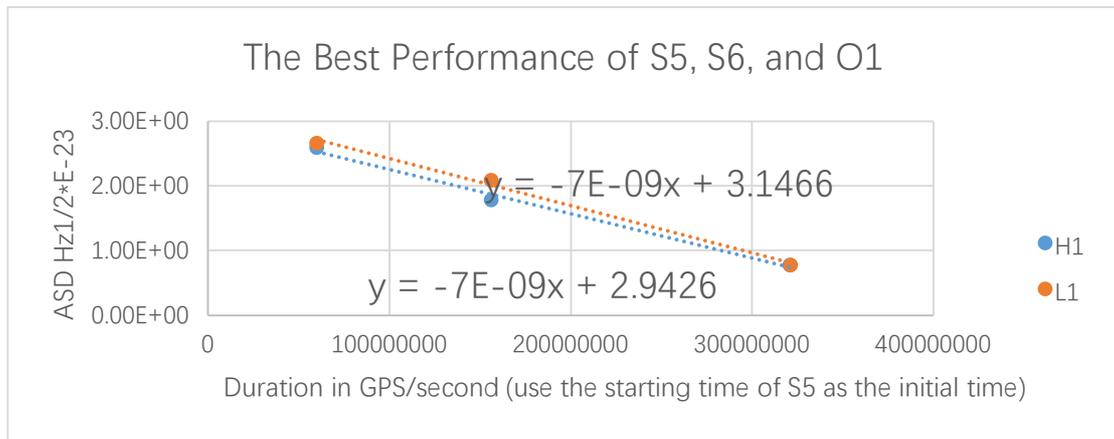


Fig.11

The Best Performance of S5, S6, and O1.

At the end of S5 and S6, the decaying exponential is flattening out, indicating that LIGO nearly reached its optimal performance in that time period. So I pick up the final ASD value of S5 and S6 as the best LIGO can do at that time. For O1, the overall pattern is more like a straight line, so the final value can also be a good indicator of LIGO's best performance.

The connection between S5, S6, and O1 is like a straight line, indicating that LIGO does improve stably over time. So it's reasonable to extend this line to extrapolate LIGO's future

performance.

### 5) The connection of “tau” between S5, S6, and O1.

The value of tau is the time at which the function is down to exponential (-1) which is about 37% of the original. At 2\*tau the function has decayed to exponential (-2), and so on. Therefore, “tau” is a good indicator of how fast LIGO can reach its best performance.

	tau/second	
	S5	S6
H1	2.50E+08	1.10E+08
L1	1.10E+08	1.00E+08

Fig.12

The tau value of H1 and L1 over S5 and S6.

I did not include the tau value of O1 here, because the performance over O1 was quite stable. For H1, there is a significant decrease in the tau value, indicating that over S6, it took LIGO, H1 half as much time as S5 to reach its maximum detector sensitivity. So a major upgrade of S6 increased the functionality of LIGO. For L1,

the tau value is quite stable over S5 and S6, indicating that a major upgrade of S6 did not significantly improved the functionality of L1, but did improve its sensitivity.

### 3. Extrapolate when the detection of primordial waves is possible

The spectral energy density of primordial waves at present time is [7]

$$h_0^2 \Omega_{\text{GW}}(f) \approx 5 \cdot 10^{-16} (H/H_{\text{max}})^2$$

, where H is the Hubble parameter during inflation and

$$H_{\text{max}} \simeq 8.4 \cdot 10^{13} \text{ GeV}$$

is the current upper bound on the energy scale of inflation. The strain amplitude of primordial wave according to this formula is much below LIGO' detectable range.

The frequencies of the primordial waves are very low, roughly one wave every 20,000 to 200,000 years. When expressed in Hertz, that's about E-12 to E-13 Hz. This is about 48 to 52 octaves below concert A (which is 440Hz). So the cosmic sounds are exceedingly

deep — way too deep for humans to hear. [8]

Since the frequency of primordial waves is exceedingly low, and LIGO generally does the best at around 150-200 Hz, LIGO will probably never reach the same performance in low frequency range as high frequency range. So several space projects are launched in order to significantly improve detector sensitivity at low frequency range. One of the promising project is LISA, [9] but even that won't get down to that lowest frequency range. We need other solutions.

## SUMMARY

The ASD values over S5, S6, and O1 show that LIGO approaches its best near the end of S5 and S6 and its performance is quite stable over O1. To look at the science runs in whole, LIGO's performance steadily improved after each major upgrade. The straight line of improvement over each science run can be extended to predict future performance.

Additionally, the best LIGO can do so far is around the 150-200 Hertz frequency range. For primordial gravitational waves, its frequency is way below that range. And LIGO generally does much worse when approaching lower frequencies. So it LIGO may never detect primordial gravitational waves. Therefore, several space projects including e-LISA will be launched to significantly decrease the noise level impeding the performance of detectors on earth.

## REFERENCES

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